EIXOS TECH
REVISTA CIENTÍFICA

Instituto Federal de Educação, Ciência e Tecnologia do Sul de Minas Gerais, Passos/MG, Brasil.
ISSN: 2359-1269
Revista Eixos Tech, v. 11, n. 5, 2024.
DOI:10.18406/2359-1269

Fredy Coelho Rodrigues

IFSULDEMINAS – Campus Passos

fredy.rodrigues@ifsuldeminas.edu.br

Marco Aurélio Alvarenga Monteiro

UNESP – Guaratinguetá

marco.monteiro@unesp.br

Demonstration and proof in the classroom: a case study involving conceptions of future mathematics teachers

Abstract

This study addresses the different types and ways of validating a mathematical conjecture in the classroom in the light of the Mathematics Education discourse. Students enrolled in the last period of the teacher training course in mathematics at IFSULDEMINAS, Campus Passos during the year 2023 participated in this investigation. In this context, the objective of the study was to investigate how future mathematics teachers understand the use and function of argument of proof in establishing a truth in the classroom. To this end, a case study was carried out, through which the data collected (questionnaire and participant observation) were examined based on the content analysis technique. As a result of the study, it was found that the majority of the subjects investigated conceive conceptual proofs as being the only type of proof capable of establishing the truth. However, the investigated subjects lack understanding and clarity in perceiving relationships between the evidence that explains and the evidence that convinces. For most study participants, both "proof" and "demonstration" are seen as synonymous terms and both have the function only of "proving that something is true".

Keywords: Mathematics Education; teacher training; empirical proof; conceptual proof; demonstration.

Demonstração e prova em sala de aula: um estudo de caso envolvendo concepções de futuros professores de matemática

Resumo

Este estudo aborda os diferentes tipos e modos de validação de uma conjectura matemática em sala de aula sob a luz do discurso da Educação Matemática. Participaram dessa investigação alunos matriculados no último período do curso de formação de professores em matemática do IFSULDEMINAS, Campus Passos durante o ano de 2023. Nesse contexto, o objetivo do estudo foi investigar como os futuros professores de matemática compreendem o uso e a função do argumento de prova no estabelecimento de uma verdade em sala de aula. Para tanto foi realizado um estudo de caso, por meio do qual os dados coletados (questionário e observação participante) foram examinados com base na técnica da análise de conteúdo. Como resultado do estudo, verificou-se que a maioria dos sujeitos investigados concebem as provas conceituais como sendo o único tipo de prova capaz de estabelecer a verdade. No entanto falta aos sujeitos investigados compreensão e clareza em perceber relações entre a prova que explica e a prova que convence. Para a maioria dos participantes do estudo tanto a “prova” quanto a “demonstração” são vistos como termos sinônimos e ambos têm a função apenas de “provar que algo é verdade”.

Palavras-chave: Educação Matemática; formação de professores; prova empírica; prova conceitual; demonstração.

Introduction

The Mathematics of Mathematics Education, in its regime of truth, "is another Mathematics, radically distinct from that seen from the perspective of the professional practice of mathematicians" (Garnica, 2002, p. 99).

Recent studies in the field of Mathematics Education have shown that educators should reconsider the exclusively "formal" way of validating a result in the classroom. These studies have highlighted the importance of also valuing other "non-formal" modes of validation that take into account the content of an explanatory argument capable of promoting a more convincing generalization (Rodrigues and Monteiro, 2021; Rodrigues, 2023; Rodrigues and Monteiro, 2024).

In the classroom context, therefore, a proof argument should provide students with an understanding (meaning) of mathematics and not just the validation of a result (Hanna, 1990; Rodrigues, 2023).

Given this context, the objective of this study is to investigate how students in a mathematics teacher training program at IFSULDEMINAS Passos Campus understand the use and function of the proof argument in establishing truth in the classroom. Underlying this understanding is a concept of "mathematical proof," which is reflected throughout the study in light of the discourse of mathematics education.

The study is justified by the importance with which the topic is currently addressed in the research literature, given that, at the curriculum level, it develops students' intellectual autonomy (Rodrigues, 2023).

In particular, in the context in which the study takes place, the investigation becomes important since the pedagogical project of the aforementioned course does not include elements that refer to the process of argumentation and proof in the training of future mathematics teachers. Therefore, the results of the investigation can provide theoretical and methodological support for the development of proactive actions to meet potential needs for reformulation/updating the course's pedagogical project.

To conduct this study, we proposed a case study to investigate and understand the different conceptions of the proof argument, as well as its function in the classroom from the perspective of the subjects. Data were collected through questionnaires and participant observation and subsequently analyzed using the theoretical lens of content analysis (Bardin, 1985).

We hope that the results obtained will provide theoretical support for rethinking mathematics teachers' pedagogical practices, thus deepening the discussion on the topic and presenting different perspectives and possibilities for using the proof argument in the classroom.

Theoretical Framework

In this topic, we will address the discourse of Mathematics Education regarding the different ways of understanding the proof argument in a mathematics classroom. The idea, therefore, is to relativize classically hegemonic positions that treat the proof argument as something exclusively logical and formal.

The different types of test levels for the classroom contexto

In the research literature, Nicholas Balacheff's (1987; 1988) studies are considered a reference for the field of Mathematics Education regarding the understanding of the different proof modes used by students to validate a conjecture.

Balacheff (1987; 1988) categorizes the types of proofs produced by students in the classroom into "pragmatic proofs" (empirical proofs) and "intellectual proofs" (conceptual proofs). Based on Balacheff's (1987; 1988) theory, pragmatic (empirical) proofs

> (...) they are related to practical knowledge and involve arguments whose justifications are empirical in nature and result from a process of observation and direct action by the student on their object of study. The validation of a proposition in this type of test fails in its general nature, since the justification is based on an explanation based on particular cases (Rodrigues and Monteiro, 2021, p.146).

As for intellectual (conceptual) tests

> (...) they are related to rational knowledge that involves arguments whose justifications result from internalized actions that flow from thought, abstract formulations and sequences of reasoning that control the entire generality of the situation. They are proofs that constitute logical-deductive discourses around the characterization of the object of study and its relationships. (Rodrigues and Monteiro, 2021, p.146).

In short, in pragmatic tests, explanations originate from concrete actions (practical testing), whereas in intellectual tests, actions are mentally internalized, and through logical-deductive discourse, the student is able to produce a generalization that can validate/explain the object of study (Balacheff, 1987; Rodrigues and Monteiro, 2024).

From pragmatic (empirical) testing to intellectual (conceptual) testing, Balacheff (1987; 1988) describes five forms (modes) of validation recognized as proof by their producers (students): naive empiricism, crucial experiment, generic example, thought experiment, and demonstration. Throughout the text, for the sake of economy of terminology, we will use the term "empirical proof" to refer to "pragmatic proof" and similarly the term "conceptual proof" to refer to "intellectual proof."

Naive empiricism: This constitutes the first and most elementary level of the hierarchy of empirical proof types. Validation occurs through the empirical testing of a conjecture in specific cases (selected without specific criteria). In this form of proof, the student, through inductive reasoning, produces an inappropriate generalization based solely on action (testing) and observation of the phenomenon (Balacheff, 1987; 1988; Rodrigues and Monteiro, 2021; Rodrigues and Monteiro, 2024).

Crucial experiment: This constitutes the second level of empirical proof. Validation occurs through a "more specific" and "judicious" empirical test, in which the student promotes an inappropriate generalization based on the analysis of a phenomenon that they believe is not a simple particular case (Balacheff, 1987; 1988; Rodrigues and Monteiro, 2021; Rodrigues and Monteiro, 2024). In other words, at this level of testing, the student verifies the conjecture for a case they consider more complex. After verifying the conjecture for this not-so-particular case, they conclude that it will be true for all other cases—that is, "if it works here, it will always work" (Balacheff, 1988, p. 219).

The first two levels or modes of proof, therefore, do not establish the truth of a conjecture (Balacheff, 1988; Triantafillou, Spiliotopoulou, and Potari, 2016) since they promote precarious or inappropriate generalization (Rodrigues and Monteiro, 2024).

For Balacheff, the language presented by the student in these first two levels of proof involves informal, poorly elaborated justification arguments, expressed mostly through the student's native language or through mathematical calculations, illustrations, graphs, and figures. According to the author, the crucial experiment begins to be considered a mode of proof to establish truth only "when it constitutes the refutation of an assertion" (Balacheff, 1988, p. 230).

Generic example: it constitutes a level of proof that marks the transition from the empirical proof argument to the conceptual proof argument (Balacheff, 1988).

> The validation method begins with a particular example of the object of study (a representative of the class) and, through operations and transformations performed on this object, explains the reasons that validate the conjecture and justify its generality. If validation involves an action focused on reasoning uniquely and exclusively focused on the particular case, then this level of proof falls into the category of empirical proof. On the other hand, if the particular case supports the expression of a type of generalizing reasoning, then this level of proof falls into the category of conceptual proof (Rodrigues and Monteiro, 2021, p. 147).

This mode of proof therefore includes "attempts to transform the properties observed in the example into abstract properties of the entire class" (Fiallo and Gutierrez, 2017, p. 4). The student, therefore, constructs a generalization (conjecture) based on a specific example seen as a representative of the class. Subsequently, they perform mental operations and formulations based on this example to demonstrate justifications and general properties that explain and validate the conjecture created (Balacheff, 1987; 1988; Rodrigues and Monteiro, 2024).

Mental experiment (conceptual mathematical proof): This constitutes one of the highest levels of proof and falls within the category of conceptual proof (Balacheff, 1988).

> The validation mode consists of justifications detached from concretization and contextualization within a particular representative. Argumentative reasoning, carried out through operations and deductions, flows through the thought that controls the entire generality of the situation. This is a level of proof where more complex cognitive constructions occur, improved discourse structuring, and better sequence of reasoning (Rodrigues and Monteiro, 2021, p. 147).

In this proof mode, the student argues more fluently in natural language and uses it as a tool for making logical deductions (Balacheff, 1987). The proof argument consists of a chain of deductive statements organized and mentally articulated with one another, and may even be supported by specific examples internalized in the mind.

The proof, therefore, is the result of operations, relationships, and articulation of mathematical concepts and ideas performed mentally and devoid of any type of concretization or practical action (Balacheff, 1987).

Rodrigues and Monteiro (2024) explain that this level of proof takes into account the content of the mathematical ideas used to validate a conjecture. Although lacking "rigor and form," the argument is generalist and explanatory and thus aligns with what Garnica (2002) calls "ethnoargumentation" for the classroom context (Garnica, 2002).

Proof (formal conceptual mathematical proof): This is considered a special type of mental experiment that does not consider "content" but rather "form" for logical reasoning. Balacheff (1988) considers this mode of argumentation to be the highest and most complex level of proof within the conceptual proof category. It is a more complex level of proof

> (...) since it presents rigor and formalization in language. (...) Proof is a type of formal argumentation that articulates the use of definitions, theorems, and logical rules of deduction and expresses a validity socially shared by the entire scientific community (Rodrigues and Monteiro, 2021, p. 147).

According to Rodrigues and Monteiro (2024), this level of proof takes into account the logical and formal structure of arguments to validate a conjecture. In the field of technology and mathematics, it therefore involves direct proofs, proofs by induction, proofs by contradiction, etc. (Rodrigues and Monteiro, 2024).

Balacheff (1988) considers conceptual mathematical proof to be less formal than formal conceptual proof (demonstration). The former takes into account the "content" to validate (Balacheff, 1988) and explain (Hanna, 1990) a conjecture, while the latter takes into account only the "formal" aspect for the purpose of validating the conjecture.

Among all the modes of proof presented, Triantafillou, Spiliotopoulou, and Potari (2016) explain that evidential arguments can be distinguished based on their "evidentiary" and "non-evidentiary" nature. According to these authors,

Non-evidentiary arguments involve empirical arguments and reasoning that capture valid arguments for or against a mathematical claim; Evidential arguments involve generic examples (e.g., particular cases seen as representative of the general case) and demonstrations (e.g., a connected sequence of statements based on accepted truths, such as axioms, theorems, definitions, and declarations). (Triantafillou; Spiliotopoulou; Potari, 2016, pp. 683-684, our translation).

In this context, the great challenge for educators is to encourage the use of proof in the classroom not only as a method to certify that something is true, but also because it is true (Hanna, 2008).

The proof argument in the context of mathematics education in the classroom

In the field of mathematics, the proof argument is characterized by the use of the "formal," "rigorous," "symbolic," and "generic" method, whose role is to "establish the truth or accuracy" of a conjecture (Rodrigues and Monteiro, 2024).

In the field of mathematics education, the use of formal proofs must be viewed with great caution, especially when teaching mathematics through argumentation and proof in the classroom (Rodrigues and Monteiro, 2023). The use of formal proofs must be reviewed and also adjusted to the classroom context, given that these demonstrations meet the needs of the mathematical community more than the needs of the mathematical community. Due to the excessive focus on "form" to the detriment of "content," they often end up being meaningless to many students (Lourenço, 2002; Rodrigues, 2023). An alternative to this, in the field of Mathematics Education, is to value the use of conceptual proofs that focus more on the "content" of the argument rather than its "form" per se (Rodrigues, 2023).

For Hanna (1990, p. 9), "a proof is valued for bringing to light essential mathematical relationships rather than merely demonstrating the accuracy of a result."

In the classroom context, therefore, the proof argument should provide students with an understanding (meaning) of mathematics and not just the validation of a conjecture. The acceptance of a conceptual proof in the classroom should take into account the understanding and establishment of mathematical meanings (content) rather than the use of the rigorous process of demonstration (form) to establish truth. A conceptual proof is valuable when it leads to understanding and helps students think more clearly and effectively about mathematics (Hanna, 2000).

Hanna sees the conceptual, non-formal proof argument as an alternative way to convince, explain, and demonstrate the validity of a conjecture. She also argues that there's a difference between "proof that proves" and "explanatory proof."

> A proof that proves only that a theorem is true; it provides only evidentiary reasons. (...) a proof that explains, on the other hand, also shows why a

> theorem is true; it provides a set of reasons that derive from the phenomenon itself. (...) a proof that proves may be based on mathematical induction or even on syntactical considerations. But a proof that explains must provide a foundation based on the mathematical ideas involved, the mathematical properties that make the mathematical theorem true (Hanna, 1990, p. 10).

The focus of explanatory proof, therefore, is understanding, not the formal mechanism of deduction. It uses mathematical properties (characterizing an entity or structure mentioned in the theorem) to explain, validate, and convince (Hanna, 1990; Hanna, 2000; Rodrigues and Monteiro, 2024).

In a formal proof argument, for example, one can be convinced that a statement is true without knowing why it is true. In other words, not every convincing proof (proof by induction) is explanatory (Hanna, 1990). Thus, the author assures, there will be no infidelity in the practice of mathematics.

> (...) if in mathematics education we concentrate as much as possible on good mathematical explanations, highlighting for students in our theorem proof the important mathematical ideas that lead to their truth. (Hanna, 1990, p.13).

The best proof, therefore, “is one that also helps us understand the meaning of the theorem to be proved: seeing not only that it is true, but also why it is true” (Hanna, 2000, p. 8). In this way, we can feel more convinced of the truth if the proof argument makes it possible to understand why a statement is true (Rodrigues and Monteiro, 2024).

Research Methodology

This study is characterized by a qualitative approach, since, according to Bogdan and Biklen (1994), the data are obtained from their own natural environment, are descriptive, and the researcher can emphasize the process rather than the product. Within the perspective of qualitative study, we chose to conduct a case study, since the objective is to investigate and understand something singular, which has value in itself (Fiorentini; Lorenzato, 2006).

The interest in the case study "focuses on what is unique and particular about it, even if certain similarities with other cases or situations later become evident" (Ludke; Andre, 1986, p. 17).

In the context of this study, the case chosen for investigation therefore involves a delimited system that encompasses the "conceptions and understandings of students graduating from a mathematics teacher training course at IFSULDEMINAS, Passos Campus, regarding the role and use of different types of tests to establish a regime of truth in the classroom."

Fiorentini and Lorenzato (2006) explain that

> A case does not simply refer to a person, a group of people, or a school. It can be any "bounded system" that presents some unique characteristics that merit special investigative investment from the researcher. (Fiorentini; Lorenzato, 2006, p. 110)

The case under investigation, therefore, "seeks to portray reality as deeply and completely as possible, emphasizing the interpretation or analysis of the object, in its context" (Fiorentini; Lorenzato, 2006, p. 110).

Within this context, the objective of the study was to investigate how students graduating from a mathematics teacher training program at IFSULDEMINAS Passos Campus understand the use and function of the proof argument in establishing truth in the classroom.

As a result of this research objective, the following research questions were outlined: 1. How do the subjects evaluated the use of different types and modes of proof in establishing the truth of a conjecture in the classroom? 2. What is the function (conception) of a proof argument from the perspective of the subjects? Below, we present the data collection instruments and the strategy used to analyze the data.

Data collection

Data were collected in the second half of December 2023 from a class (8th semester) of Mathematics undergraduate students at IFSULDEMINAS, Passos Campus, during the Knowledge Production II class taught by the study's lead author.

Students who agreed to participate in the study signed an Informed Consent Form (ICF) expressing their agreement and consent to the risks and benefits of participating in the study. Participants were also guaranteed the right to confidentiality and the anonymous use of the collected data. Therefore, data collection ensured that students were protected by the law governing the ethical principles of research involving human subjects.

During data collection, the researchers used a mixed-method questionnaire (Appendix A). The questionnaire consisted of five questions: one closed-ended and the other four open-ended.

In the first question, the research subjects were presented with various types and modes of evidence to assess their perception of the evidentiary nature of the evidence presented. The collection of evidence (five different types of arguments) presented in question 1 of the questionnaire was developed by the study authors and took into account the different types and modes of validation proposed by Balacheff (1987; 1988).

The evidence was therefore hierarchically classified in the following order:

Example 1 – empirical evidence at the level of naive empiricism.

Example 2 – empirical evidence at the level of a crucial experiment.

Example 3 – conceptual evidence at the level of a generic example.

Example 4 – conceptual evidence at the level of informal thought experiment.

Example 5 – conceptual evidence at the level of formal thought experiment. Furthermore, the other four questions in the questionnaire were open-ended and intended to gather data on the function (conception) of the proof argument, the assessment of its explanatory character, and the most convincing type of proof in the students' view.

Concurrently with the questionnaire administration, the researcher (first author of the study) acted as a participant observer, collecting additional data and clarifying doubts regarding the questionnaire questions.

During the questionnaire administration (2 hours), the researcher (first author of the study), practicing "participant observation" (Ludke and Andre, 1986), sought closer contact with the research subjects, clarifying doubts and prompting reflections on the questions posed in the questionnaire.

Observation, therefore, in its descriptive part, involves a detailed record of what occurred and documented in notes or a field diary (Fiorentini and Lorenzato, 2006). It allows for: describing the study site; characterizing special events; describe the activity performed and the behaviors of the researcher and the subjects under investigation. Regarding the reflective aspect, observations evoke in the researcher: "speculations, feelings, problems, ideas, impressions, preconceptions, doubts, uncertainties, surprises, and disappointments" (Ludke and Andre, 1986, p. 31).

In short, the data collected essentially comprise the information gathered by the research questionnaire as well as additional information gathered through participant observation and documented in field notes.

Data analysis strategy

To analyze the data, we used the "Content Analysis" technique (Bardin, 1985) to better understand the content of the information emerging from the information collected through the questionnaire and observations transcribed into field notes.

Fiorentini and Lorenzato (2006) explain that Content Analysis takes into account "the words used in the responses, the ideas or opinions expressed, and the interpretations and justifications presented" (Fiorentini and Lorenzato, 2006, p. 137). The analysis of the information becomes:

> (...) a laborious and meticulous process that involves multiple readings of the available material, trying to find units of meaning or patterns and regularities in it, and then grouping them into categories. The search for this organization is generally guided by the investigative question and the objectives of the study. (Fiorentini and Lorenzato, 2006, p.133).

Categorization, therefore, consists of a "process of classifying or organizing information into categories, that is, into classes or sets that contain common elements or characteristics" (Fiorentini and Lorenzato, 2006, p. 134).

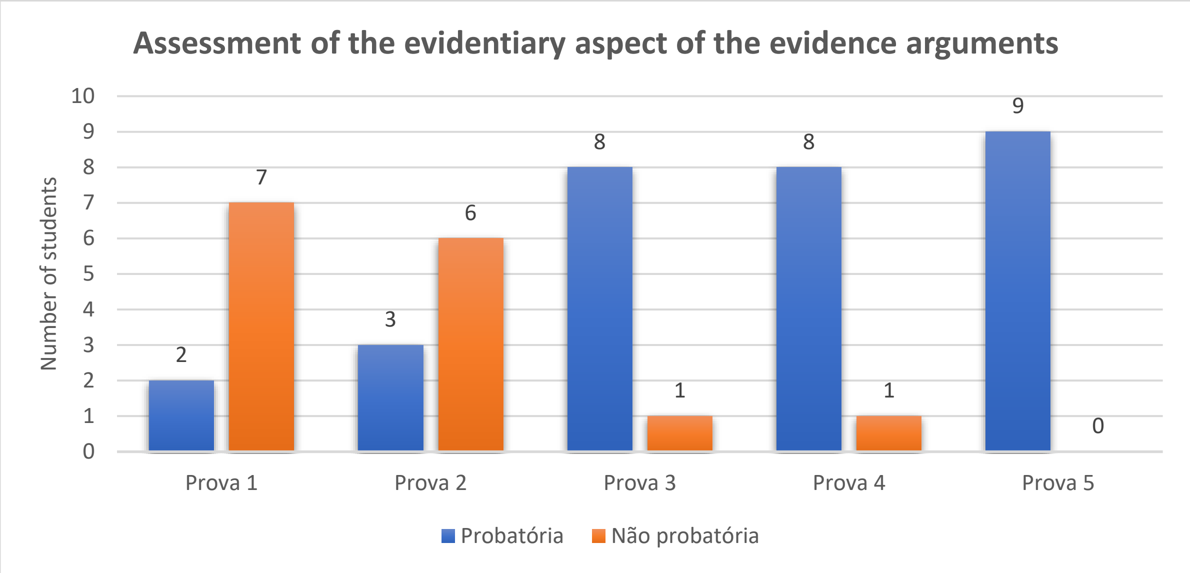
During data analysis, a type of category defined as "emergent" was used, obtained "through an interpretative process, directly from the field material" (Fiorentini and Lorenzato, 2006, p. 135). Separating information into units of meaning, through categories, facilitates the comparison of information and the perception of relationships, patterns, and regularities, thus favoring the "iterative construction of an explanation" for the phenomenon under study (Fiorentini and Lorenzato, 2006, p. 139).

Results and discussion

Nine (9) graduating students enrolled in the 8th semester of the Mathematics Bachelor's degree program at IFSULDEMINAS, Passos Campus, participated in the study.

The first question of the research questionnaire presented five types of tests with varying levels of complexity. The students were then asked to evaluate and judge the evidentiary nature of each of the different tests presented. The results are presented in Figure 1 below.

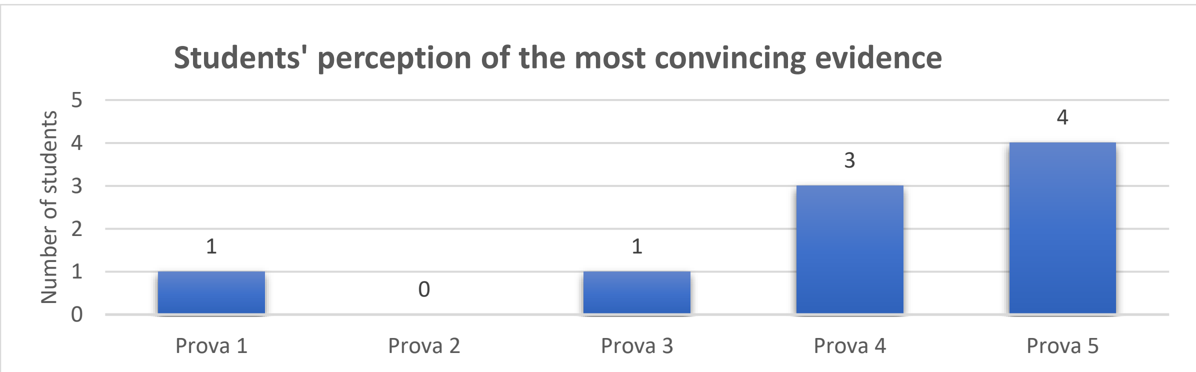
Figure 01: evidentiary or non-evidentiary evidence



Source: Research data (2023).

For most students, the arguments presented in test 1 and test 2 were not considered probative. However, the arguments presented in test 3, test 4, and test 5 were considered probative. In general, most students successfully correctly associated the empirical proof arguments (test 1 and test 2) with a proof without probative purpose. Likewise, they associated the generic proof arguments (test 3, test 4, and test 5) with a probative proof (Balacheff, 1987; Triantafillou, Spiliotopoulou, and Potari, 2016; Rodrigues and Monteiro, 2024). The result also provides evidence that for a minority of students (no more than three), it is still unclear which type of proof (empirical or generic) is considered valid to prove the truth of a conjecture. In the second question of the questionnaire, students were asked, "Which evidentiary argument do you find most convincing? Why?" The results are presented in the graph in Figure 2.

Figure 2: The most convincing evidentiary argument



Source: Research data (2023).

Most of the students surveyed (8) considered the conceptual proofs (test 3; test 4; test 5) to be the most convincing, for several reasons: "more generic," "better quality in terms of their elaboration," "more solid," and "greater strength in validating the proposed conjecture" when compared to the empirical proofs (Balacheff, 1987; Rodrigues and Monteiro, 2024).

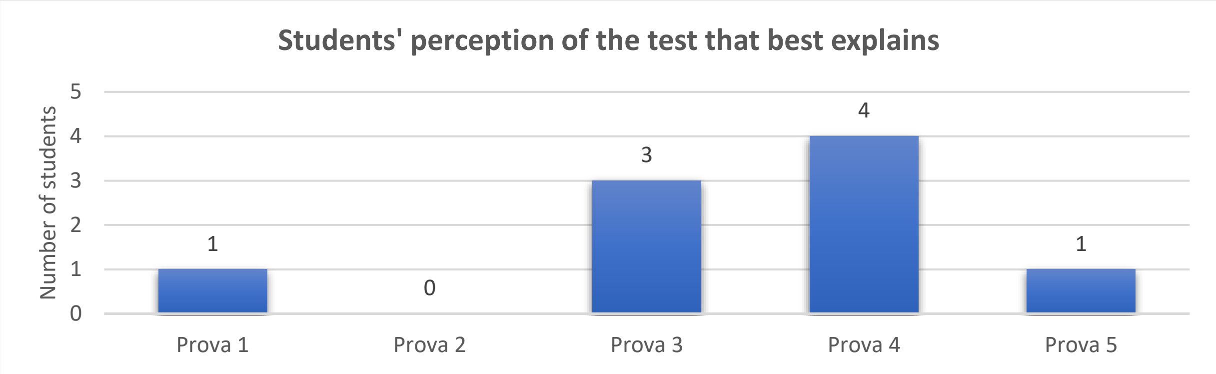
In particular, half of these students (4) considered the formal conceptual proof (test 5) to be the most convincing argument and with the greatest potential for validation. This is because, in their understanding, the argument used was based on the principle of induction, a validation method commonly used in the field of professional mathematics to establish truth (Balacheff, 1987; Rodrigues and Monteiro, 2024).

The other half of the subjects (4) highlighted the value of conceptual proofs at the level of mental experiment (proof 4) and generic example (proof 3) for better convincing the truth (Balacheff, 1987; Rodrigues and Monteiro, 2024). In these students' view, these proofs provided greater clarity and conviction regarding the understanding of the content and the genesis of the conjecture.

In the field of Mathematics Education, these last two proof modes (proof 3; proof 4) have been highlighted for the context of valuing different ethnoarguments (Garnica, 2002) in the classroom. This is because these arguments are also considered valid for purposes of proof and have the advantage of being more explanatory and, consequently, more convincing proofs (Hanna, 1990) when compared to conceptual proofs at the formal level (Balacheff, 1987; Rodrigues and Monteiro, 2024).

Only one student highlighted empirical evidence at the naive empiricist level (evidence 1) as the most convincing proof argument. This student believed that simply empirically testing the conjecture alone is capable of establishing the truth of the proposed conjecture, which is a mistake (Balacheff, 1987; Rodrigues and Monteiro, 2024). In the third question of the questionnaire, students were asked, "Which proof argument do you consider more explanatory? Why?" The results are presented in the graph in Figure 3.

Figure 03: The most explanatory proof argument



Source: Research data (2023).

For most of the participants, the informal conceptual proof arguments (tests 3 and 4) were considered more explanatory than the formal conceptual proof argument (test 5). Seven students considered tests 3 and 4 to be more detailed in the "emergence," "explanation," "justification," "elucidation," and "clarification" of why the conjecture is valid and true. This understanding is consistent with the ideas of Hanna (1990) and Rodrigues and Monteiro (2024), who argue that the best proof argument is one that simultaneously proves that something is true and explains why it is true.

The results obtained, when compared to the result of question 2, show that the proof considered the most convincing, due to the use of the scientific method, is not always considered the most explanatory.

In particular, the conceptual proof at the non-formal thought experiment level (test 4) stood out as the most explanatory proof argument for most students (4), followed closely by the conceptual proof at the generic example level (test 3), which was preferred by three (3) students. The difference between one argument and the other lies in the type of reasoning used to establish a generalization. In test 3, the reasoning is inductive, and in test 4, the reasoning used is deductive (Balacheff, 1987; Rodrigues and Monteiro, 2024). For Rodrigues and Monteiro (2024), Proof 4 possesses a greater degree of sophistication, elaboration, strength, and solidity than Proof 3.

A single student expressed confidence in the explanatory power of Proof 1. For this student, empirical proof at the naive empiricist level is sufficient to explain the validity of the conjecture, which is a mistake according to Balacheff (1987) and Rodrigues and Monteiro (2024). This student is the same one who cited empirical proof as the most convincing argument in question 2 of the questionnaire. It is therefore assumed that this student does not yet understand the role of a proof or demonstration in validating an idea. The student who cited the conceptual proof argument at the formal level (Proof 5) as the best-explaining proof simply states that he trusts the principle of induction; however, he performs some tests later to convince himself of the use of the method. This student's testimony therefore corroborates the idea that "formal proof" or "rigorous mathematical proof" in some situations proves but does not explain (Hanna, 1990; Rodrigues and Monteiro, 2024).

In the fourth question of the questionnaire, study participants were asked "what is the function of a mathematical proof in the classroom?" The results are presented in Table 1 through categories that best summarize and organize the responses of the subjects studied.

Table 01: Conceptions about the function of mathematical proof in the classroom

Category	nº of students	Answers
Prove it is true	5	R_1: "to prove that a statement is true"
		R_2: "establish unconditional truth"
		R_3: "to show that an idea is true"
		R4: "demonstrate the generality of the case"
		R5: "demonstrate the validity of a theorem"
Prove and explain why it is true	1	R6: "Show that the conjecture is true through explanations"
Convince about the truth from a test	1	R7: "The purpose of the test is to test a formula to convince the student that it works and that they are not held hostage by rote memorization."
Assess knowledge	2	R8: "Evaluate all student thinking styles to achieve the same final result." R9: "Evaluate knowledge through demonstration."

Source: Research data (2023).

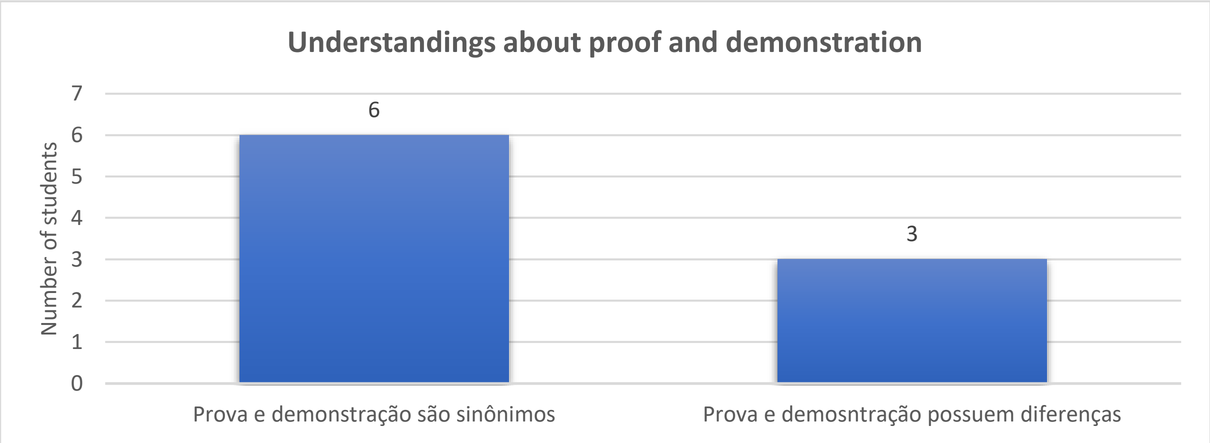
Based on what is presented in Table 1, the responses were grouped into four (4) categories: "prove that it is true"; "prove and explain why it is true"; "convince of the truth through a test"; "evaluate knowledge."

For the majority of students (5), the test has an exclusively evidentiary function, that is, to prove that a conjecture is true (Triantafillou, Spiliotopoulou, and Potari, 2016). On the other hand, two (2) students highlighted the function of the test as a tool for assessing knowledge. That is, in their statements, the students, therefore, open up the possibility that this assessment allows for the identification of the type, level, and quality of the evidentiary argument produced (Balacheff, 1987; Rodrigues and Monteiro, 2024). The role of proof as a test element for convincing the truth was cited by a single student, who mistakenly maintained a belief in a very reductive conception of proof that is limited to testing and verifying the conjecture to establish the truth (Balacheff, 1987; Rodrigues and Monteiro, 2024). Finally, a single student emphasized the explanatory function of proof during evidentiary proceedings (Hanna 1990; Rodrigues and Monteiro, 2024).

From the perspective of the student producing a mathematical proof in class, we therefore emphasize the importance of the argument playing both a probative and an explanatory role. After all, a good evidentiary argument is one that proves that a conjecture is true and also explains why it is true (Hanna, 1990; Rodrigues and Monteiro, 2024). From the perspective of the mathematics teacher, carefully evaluating students' proof arguments allows us to identify the type of proof produced by the student as well as their level of proof. Furthermore, assessing knowledge through proof allows the teacher to verify the quality and soundness of the proof based on criteria established by Rodrigues and Monteiro (2024).

In the final question of the questionnaire, students were asked, "In your opinion, is there any difference between proof and demonstration? If so, please explain." The results are presented in the graph in Figure 4.

Figure 04: proof and demonstration



Source: Research data (2023).

According to the graph, most students (6) understand that "proof" and "demonstration" are synonymous terms, thus contradicting Balacheff (1987) and

Rodrigues and Monteiro (2024), who describe and characterize demonstration as a special and particular type of conceptual proof.

Three (3) students responded that there is indeed a difference between the two terms and presented their justifications.

I don't know how to accurately define the terms "proof" and "demonstration," but I understand that proof is linked to something more specific, while demonstration, on the other hand, is linked to something broader and more general. (Student 1, research data).

Proof is associated with an example, and demonstration is related to more general knowledge, involving the use of mathematical rules and concepts to explain one's reasoning. (Student 2, research data).

Proof refers to a particular case of demonstration, which is broader and uses in-depth resources. Demonstration is a set of proofs. (Student 3, research data).

The justifications, therefore, demonstrate a very reductive understanding of the term "proof" when compared to the term "demonstration." It is as if the power of proof were limited to verifying and testing empirical cases.

Given the evidence, it became clear that conceptual proof (Balacheff, 1987) is not as well understood by the students, just as demonstration as a type of conceptual proof is also unfamiliar to all of them.

Given the students' points, we believe that improving the quality of the proof argument depends on their understanding of what it means to prove and demonstrate, as well as understanding that a conceptual proof (Balacheff, 1987) that explains (Hanna, 2000) has the power to prove as much as a mathematical demonstration (formal conceptual proof).

Final considerations

Throughout this study, we investigated how students in a mathematics teacher training program at IFSULDEMINAS Passos Campus understand the use and function of the proof argument in the classroom. The collected data were analyzed and compared with mathematics education discourse regarding the different ways of understanding the proof argument in a mathematics classroom.

Overall, the results revealed that most of the participants had a good understanding of aspects related to the "nature" and "type" of proof used to establish truth in the classroom. Most participants agreed that empirical proof based on specific cases cannot be used for validation purposes. On the other hand, generic proof arguments, whether "formal" or "informal," and lacking concrete concreteness in specific cases can be used for validation purposes.

During the analysis of questions 2 and 3 of the research questionnaire, for example, it became clear that the students studied lacked clarity in understanding the meanings of "convincing proof" and "explanatory proof," as well as the possible relationships between them. This was evident when most students identified proof argument 5 (proof by induction) as the "more convincing" and "less explanatory" argument, compared to proof argument 4, which they identified as "more explanatory" and "less convincing." This lack of clarity contrasts, therefore, with Hanna's (1990) understanding that the explanatory nature of a proof makes it more convincing.

When asked about the function of a mathematical proof in the classroom, most students indicated that its function is linked to "proving that something is true." However, it is important to emphasize that in the classroom, the mathematics teacher should also highlight and value other functions of mathematical proof, namely, "proving and explaining why it is true"; "testing to convince oneself of the truth" and "evaluating students' knowledge."

Most students conceive of the terms "proof" and "demonstration" as synonymous. A minority of the subjects studied, however, understand "proof" as a particular type of mathematical "demonstration." This misconception about the proof argument contradicts the mathematics education discourse that classifies "demonstration" as a special type of formal conceptual "proof." In this sense, knowing how to better differentiate and understand these terms allows mathematics teachers to assess the levels of proof produced by students and, from there, work with them to construct more elaborate proofs of a probative and explanatory nature.

The results, therefore, allow us to conclude that conceptual proof (Balacheff, 1987) is not well understood by these students, just as demonstration as a type of conceptual proof is also unfamiliar to them. It is imperative to emphasize that improving the quality of proof arguments depends on students' understanding of what proving and demonstrating are, as well as understanding that a conceptual proof that explains has the same power of proof as a mathematical demonstration (formal conceptual proof).

We recommend that further studies be conducted using other teacher training contexts as a reference to deepen and generalize our reflections on the topic under investigation. Investigating and reflecting on the different beliefs and misconceptions regarding the use of proof arguments in the classroom provides educators with theoretical and methodological support to rethink and transform educational practices.

In the context of the Mathematics Undergraduate Program where the study was conducted, the results obtained and the reflections presented advocate and suggest the inclusion of a course on "argumentation and proof in teacher training" within the course's Pedagogical Project, aiming to contribute to the teaching and learning of the different types and modes of argumentation/proving in the classroom.

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